Control strategies of the Limited Bandwidth Hydro-pneumatic Active Suspension for Road Vehicle

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Abstract— **The active suspension required energy input levels and the high component costs. In this work, the Limited bandwidth hydro-pneumatic active suspension (LBA) is used and it has all the advantages of active system except that the actuator is limited at 6 Hz bandwidth, thus making it economical on power consumption and cost. The aim of this study is to develop four degrees of freedom (DOF) half vehicle model including the Limited-Bandwidth hydro-pneumatic active suspension system. The LQR and Fuzzy Logic Control (FLC) are used to evaluate the vehicle ride performance for Limited bandwidth hydro-pneumatic active suspension. The result indicated that the limited-bandwidth Hydro-pneumatic active suspension with LQR gives better ride performance compared with the passive suspension system. On the other hand, the FLC improved the vehicle ride performance in terms of front and rear body acceleration by 4% and 7.5% respectively compared with limited-bandwidth Hydro-pneumatic with LQR. The power demand for limited-bandwidth Hydro-pneumatic with LQR and FLC are evaluated and discussed.**

***Index Terms*— Limited bandwidth hydropneumatic active suspension, Fuzzy Logic Control (FLC), LQR, Anti-Lock Braking System (ABS)**

I. INTRODUCTION

The main idea of the limited-bandwidth Hydro-pneumatic active suspension is to use the active device to control the system dynamics around the body resonance and to allow passive elements to exercise suitable control for higher frequency components. The active suspension systems offer the best overall performance, but are considerably impractical, because of extremely high cost involved. The limited-bandwidth Hydro-pneumatic active suspension is more practical and performs nearly as well [1-3]. Other researcher [4] developed a methodology for the design and evaluation of a slow-active vehicle suspension system. They designed an optimal multivariable controller for a full car model in terms of seven degrees-of-freedom. This controller requires a linear quadratic regulator form with supplementary states to add integral action. Their results showed that the slow-active systems offer significant improvements in controlling body resonances. This system consumes low power compared to active systems. This fact is recorded in the vehicle literature over the past few years [5].

Several control strategies for active and slow active are used [6-9]. Recently, a Model Reference Control (MRC) strategy for active suspension System is developed [10]. The MRC technique utilizes both wheelbase preview concepts and suspension look-ahead preview and the MRC methodology depended on an ideal hybrid skyhook-groundhook concept and also MRC technique utilizes 8 Proportional-Integral-Derivative controllers for body and each wheel control. Their results showed that, the proposed MRC strategy with the PID controllers are able to track the performance of the ideal hybrid skyhook-groundhook system, and provided significant improvements in both ride comfort and road holding. It is clear from the previous literature that more investigations of limited-bandwidth Hydro-pneumatic active suspension are required. In this work, the LQR and Fuzzy Logic Control (FLC) are used to evaluate the vehicle ride performance for Limited bandwidth hydro-pneumatic active suspension (LBA). Also, the power demand for LBA with LQR and FLC are evaluated and discussed.

II. VEHICLE MATHEMATICAL MODEL

A. *Equation of motion*

The four degrees of freedom half vehicle model used is shown in Fig. 1.

The equations of motion can be derived by applying Newton’s second law for the vehicle body and wheel masses respectively as follow [1, 11 ,13]:

$$M_b\ddot{Z}_b = -F_{sf} - F_{sr} \tag{1}$$

$$I_b\ddot{\theta}_b = L_f F_{sf} - L_r F_{sr} - h_f\left(F_{xf} - F_{rf}\right) - h_r\left(F_{xr} - F_{rr}\right) \tag{2}$$

$$M_{wf}\ddot{Z}_{wf} = -K_{tf}\left(Z_{wf} - Z_{of}\right) + F_{sf} \tag{3}$$

$$M_{wf}\ddot{Z}_{wr} = -K_{tr}\left(Z_{wr} - Z_{or}\right) + F_{sr} \tag{4}$$

Where,

$$h_f = H + \left(Z_{bf} - Z_{wf}\right) + \left(Z_{wf} - Z_{of}\right) \tag{5}$$

$$h_r = H + \left(Z_{br} - Z_{wr}\right) + \left(Z_{wr} - Z_{or}\right) \tag{6}$$

For passive suspension system the suspension dynamic forces can be written as the following:

$$F_{sf} = K_{sf}\left(Z_{bf} - Z_{wf}\right) + C_{sf}(\dot{Z}_{bf} - \dot{Z}_{wf}) \quad (7)$$

$$F_{sr} = K_{sr}(Z_{br} - Z_{wr}) + C_{sf}(\dot{Z}_{br} - \dot{Z}_{wr}) \quad (8)$$

(1)	Oil tank	**(6,7)**	Font and rear gas springs
(2)	Pump	**(8,9)**	Front and rear throttle valves
(3)	Main accumulator	**(10,11)**	Front and rear suspension struts
(4,5)	Front and rear direction control valves		

Where:

$\mathbf{Z_b}$	Body vertical displacement at center of gravity
$\mathbf{Z_{bfr}}$	Front and rear body vertical displacement
$\mathbf{Z_{of,r}}$	Front and rear road input
$\mathbf{Z_{wf,r}}$	Front and rear wheel vertical displacement
$\boldsymbol{\theta}$	Body pitch angle

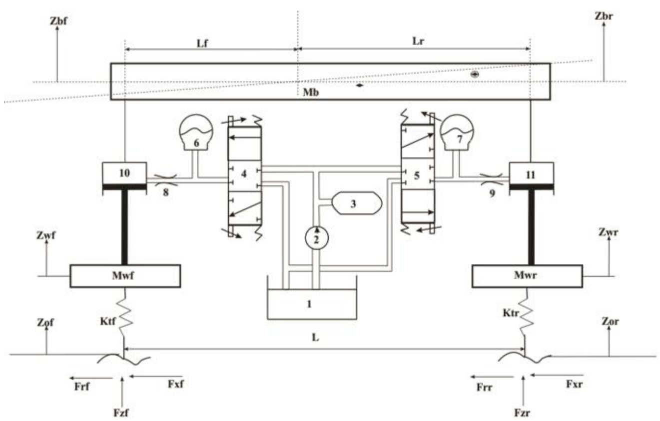


Fig. 1 Half vehicle model with limited bandwidth hydro-pneumatic active suspension system

A limited bandwidth hydropneumatic active suspension operates to control the vehicle ride characteristics over the lower frequency range in particular up to 6 Hz. For limited bandwidth hydropneumatic active suspension system the dynamic forces of the suspension can be written as the following:

$$F_{sf} = A_{st}.\left[P_{gstf}({V_{gstf}}^{\gamma}/((V_{gstf} - V_{gf})^{\gamma}) - P_{gstf} + C_1.A_{st}\left(\dot{Z}_{bf} - \dot{Z}_{wf}\right)\right] \quad (9)$$

$$F_{sr} = A_{st}.\left[P_{gstr}({V_{gstr}}^{\gamma}/((V_{gstr} - V_{g})^{\gamma}) - P_{gstr} + C_1.A_{st}\left(\dot{Z}_{br} - \dot{Z}_{wr}\right)\right] \quad (10)$$

Most of the studies proved that, the vehicle model can be reduced in the two-dimensional model seen in Figure 1 as long as the vehicle speed is constant. It can be observed for long wavelengths, the coherence between the left and right tracks is likely to be high, and the road surface may be regarded as cylindrical. Consequently, the two sides of the vehicle will behave in a similar fashion. Also, nothing that for short wavelengths the motions excited in the vehicle will mostly involve wheel hop. Little body motion will occur, and left and right will interact very little [11, 12].

B. *Road input and vehicle parameters*

The road input is presented using the following equation;

$$\dot{R}_o(t) = -2\pi.f_o.R_o(t) + \sqrt{G \cdot G_o(t) \cdot v(t)}\,w(t) \quad (11)$$

In this study, R_o, Go, v, $w(t)$ and f_o are the displacement of road input, the road input roughness coefficient, the driving speed, zero-mean Gaussian white noise, with its intensity1 and the low cut-off frequency (0.01 Hz). The road roughness coefficients and the half vehicle parameters used for the calculations are shown in Table (1) and Table (2) respectively [12].

TABLE I
ROAD ROUGHNESS COEFFICIENT

Road roughness	Class
$256*10^{-6}$	B
$512*10^{-6}$	C

The vehicle parameters are shown in TABLE II.

TABLE II
VEHICLE PARAMETERS

Parameter	Value/Unit	Description
M_b	690 kg	Body mass
M_{wf}, M_{wr}	40.5 and 45 kg	Front and rear wheel masses
I_p	$1222\ \mathrm{kg.m^2}$	Body moment of inertia
K_{sf}, K_{sr}	17 and 22 KN/m	Front and rear spring stiffness
C_{sf}, C_{sr}	1.5 KN.s/m	Front and rear damping coefficient
K_{tf}, K_{tr}	192 KN/m	Front and rear tire stiffness
H	0.328	The height of vehicle C.G from the road surface
L_f, L_r	1.25 and 1.51 m	Distance from C.G to front and rear axles
γ	1.4	Gas constant
A_{st}	$8.04 * 10^{-4}\ \mathrm{m^2}$	Strut area

III. CONTROLLERS

Two different control algorithms have been advanced to the limited bandwidth active suspension system. The first strategy is based on optimal control theory using limited state feedback concept, while the second strategy is related to FLC. In order to design the limited bandwidth active controllers, it is assumed that the vehicle has body and wheel vertical acceleration sensors at each corner to support the control algorithm with the body and wheel vertical accelerations states. Also, the vehicle has a suspension travel sensor at each corner to support the control algorithm with the suspension travel states.

A. *Optimal Control theory*

Optimal control theory that interested in operating the dynamic system at least cost. The theory is a part of applied mathematics which has apriority to get the control law for applying it on the dynamical system through time period until optimizing the objective function. A set of linear differential equations use to describe system dynamics. In addition to, it has various applications in both engineering and science. For instance, the dynamical system can be an automobile with controls related to vertical automobile dynamics, and the objective of that may be to improve ride comfort with minimum power demand and maintain SWS. Also, possible that dynamical system may be spacecraft, with its objective to reach moon with the controls related to rocket thrusters [11]. Optimal control can be seen as a control strategy in control theory. One of the major results in this theory is which the solution has been provided by the linear quadratic regulator.

As known, applying the full state feedback control concept for the LQR controllers is unpractical due to the difficulties in measuring the road input. Therefore, the limited state feedback concept is selected to derive the feedback LQR control law shown in equation (12) [10].

$$\mathrm{P_{gdi}} = \mathrm{K_{1i}}\ddot{\mathrm{Z}}_{\mathrm{bi}} + \mathrm{K_{2i}}\ddot{\mathrm{Z}}_{\mathrm{wi}} + \mathrm{K_{3i}}(\mathrm{Z_{bi}} - \mathrm{Z_{wi}}) \qquad (12)$$

Where $K_{1..3,}$ are the LQR control gains.

B. *Fuzzy Logic Control*

Fuzzy-Logic is used to design a practical and cost-effective controller for the limited bandwidth active suspension system. FLC is considered as one of the smartest control methods and it also, presents different unparalleled features which make from FLC is a best choice for many control issues. Non-linear system which is impossible to represented by mathematically can be controlled by FLC. It also does not need precise or noise free input. It can be programmed to control the system even if a feedback sensor is damaged. The control output is a smooth function in spite of a wide extent of input variations. so, any sensor data that supplies many indications of any systems actions and reactions are suitable. For all of that, FLC let the sensors to be inexpensive and inaccurate so this keeps the overall system cost and intricacy low. Because of these advantages, the Fuzzy-Logic control is used to develop a practical and cost effective the limited bandwidth active suspension system controller. The FLC controller requires the body vertical acceleration and the suspension velocity signals as a controller inputs, which leads to cost improvement in the overall system as the wheel vertical acceleration signals/sensors not required. The output signals are the demand P_{gd} at each corner. The rule base and interface engine are formed with Mamdani-Type of fuzzy inference, while the defuzzification process is based on center of area method. The rule base of the developed FLC algorithm is shown in table III.

TABLE III
SLOW-ACTIVE FLC RULE BASE

P_{gd}		Front or Rear Suspension Velocity						
		NB	NM	NS	ZE	PS	PM	PB
Front or Rear Body Acceleration	NB	NB	NB	NB	NB	NB	NB	NB
	NM	NM	NM	NM	NM	NM	NM	NM
	NS	NS	NS	NS	NS	NS	NS	NS
	ZE	ZE	ZE	ZE	ZE	ZE	ZE	ZE
	PS	PS	PS	PS	PS	PS	PS	PS
	PM	PM	PM	PM	PM	PM	PM	
	PB	PB	PB	PB	PB	PB	PB	PB

The direction control valve is considered as the first order transfer function shown in Equation 13 has been used for simulating the dynamics of slow active control valve

$$Q_{Ai}(s) = \frac{1}{(t_d(S)+1)} Q_{AD_i}(s) \qquad (13)$$

Where, $Q_{AD_i}(s)$ is the Laplace transform of the desired flow rate $Q_{AD_i}(t)$, while $Q_{Ai}(s)$ is the actual flow rate whose time domain form is $Q_{Ai}(t)$, and $t_d(s)$ is the Laplace transform of the time delay constant. The following performance index shown in Equation 14 is selected for the optimization process.

$$\mathbf{J} = \mathrm{q_1\ RMS_{Acc_f}} + \mathrm{q_2\ RMS_{Acc_r}} + \mathrm{q_3\ RMS_{DTL_f}} + \mathrm{q_4\ RMS_{DTL_r}} + \mathrm{q_5\ RMS_{SWS_f}} + \mathrm{q_6\ RMS_{SWS_r}} \qquad (14)$$

As shown in equation (14), the performance index is the weighted sum of the Root Mean Square (RMS) of the body vertical accelerations, dynamic tire loads and the suspension deflections. All the performance index components are normalized with the passive suspension performance and weighted by the weighting parameters $q_{1\text{-}.6}$ to emphasize the importance of each component [12].

IV. RESULTS AND DISCUSSIONS

A. *Comparison between passive suspension, slow active with optimal control and FLC using road input (class B)*

In the limited bandwidth hydro-pneumatic active suspension an actuator with 6Hz bandwidth is used to control the suspension. The vehicle has been simulated over a road input (Class B) with constant vehicle speed 100 km/hr. Fig. 2 and Fig. 3 show the power spectral density of the body accelerations, suspension working space and dynamic tyre load for the passive suspension system, limited bandwidth hydropneumatic active suspension systems with LQR and FLC. The comparisons are made in terms of power spectral density. In Fig. 2 shows the body bounce, pitch accelerations, vehicle body at CG and rear body acceleration for the passive system and limited bandwidth hydropneumatic active suspension with LQR and

FLC. It can be noticed that the ride performance of the limited bandwidth hydropneumatic active suspension with FLC gives worthwhile improvements than the passive suspension system in terms of body acceleration and DTL. Also, the system gives better ride performance than the limited bandwidth hydropneumatic active suspension with LQR. These improvements are clearly seen in the frequency range up to 6.0 Hz.

Fig. 3 shows the rear suspension working space and dynamic tyre load for the passive system and limited bandwidth hydropneumatic active suspension with LQR and FLC. It can be seen that two point were emerge; (i) the rear suspension working space and dynamic tyre load of the limited bandwidth hydropneumatic active suspension with FLC gives better improvements than the passive suspension system and limited bandwidth hydropneumatic active suspension with LQR; (ii) there is improvements in terms of dynamic tyre load around the unsprung mass resonance frequency is observed with both limited bandwidth hydropneumatic active suspension system with FLC and LQR.

The summary of ride performance improvements in terms of root means square of front and rear body accelerations using road input roughness (Class (B)) is shown in Table IV. The percentages reduction of front and rear body accelerations for limited bandwidth hydro-pneumatic active suspension with LQR compared with passive suspension are 12.3% and 16.1% respectively. On the other hand, the percentages reductions of front and rear body accelerations for limited bandwidth hydro-pneumatic active suspension with FLC compared the same system with LQR are 4% and 7.5% respectively. Furthermore, this percentage reduction for the same system is increased to 16.6% in case of pitch acceleration.

TABLE IV
RIDE PERFORMANCE IMPROVEMENTS WITH ROAD INPUT (CLASS B)

System Performance		Passive Suspension	LBA with LQR	LBA with FLC
Front	Acc, m/sec^2	0.5849	0.5132	0.4925
	SWS, m	0.00556	0.006459	0.006996
	DTL, N	404. 6	430.5	448.6
Rear	Acc, m/sec^2	0.7507	0.6299	0.5824
	SWS, m	0.005936	0.006346	0.006729
	DTL, N	419.4	454.2	488.2
CG Body, Acc. m/sec^2		0.6515	0.5618	0.5298
Pitch Acc., m/sec^2		0.09763	0.06549	0.05465

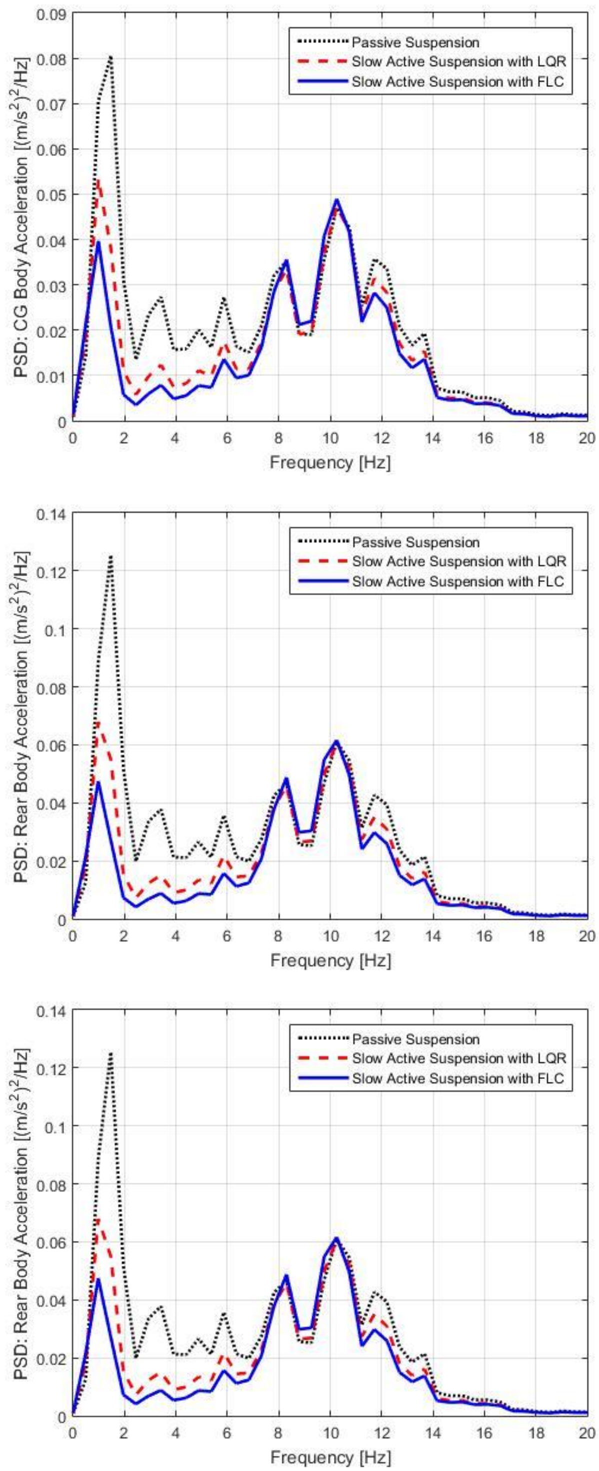


Fig. 2. Power spectral density of body C.G bounce, pitch accelerations and rear body acceleration for passive and limited bandwidth hydropneumatic active suspension with LQR and FLC using road input (class B)

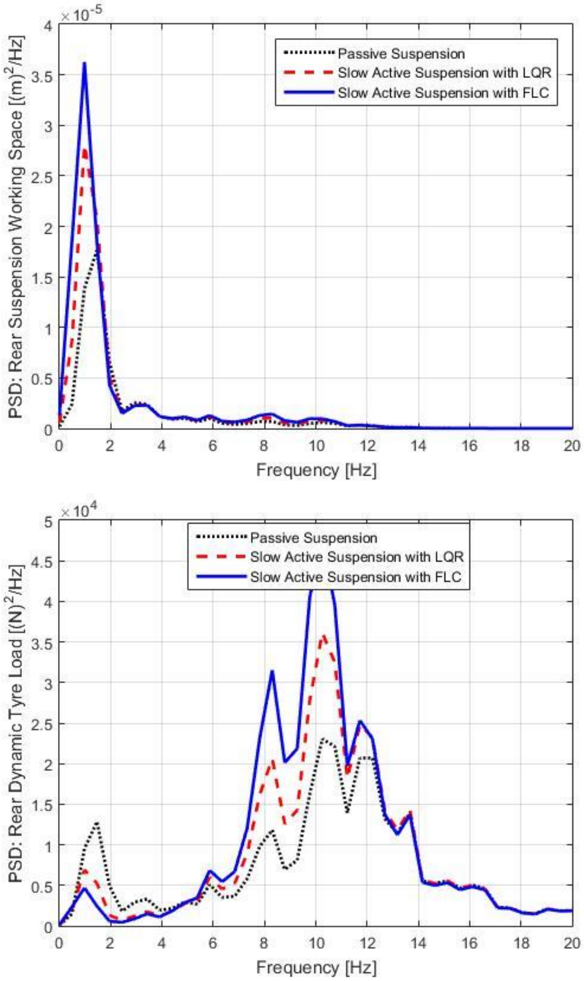


Fig. 3. Power spectral density of rear suspension working space and dynamic tyre load for passive and limited bandwidth hydropneumatic active suspension with LQR and FLC using road input (class B)

B. Comparison between passive suspension, slow active with optimal control and FLC using road input (class C)

Fig. 4. shows comparison of the body bounce, rear body acceleration and pitch accelerations at the vehicle body CG for the passive system, limited bandwidth hydropneumatic active suspension systems with LQR and FLC in terms of power spectral densities. The power spectral densities curves showed a clear improvement around body resonance peak in the body CG, pitch and rear body accelerations of the limited bandwidth hydropneumatic active suspension system with FLC in comparison with the passive suspension system and the limited bandwidth hydropneumatic active suspension system with LQR, in the overall frequency range. Also, a very small improvement around the unsprung mass resonance peak is observed for both limited bandwidth hydropneumatic active suspension system with FLC and LQR.

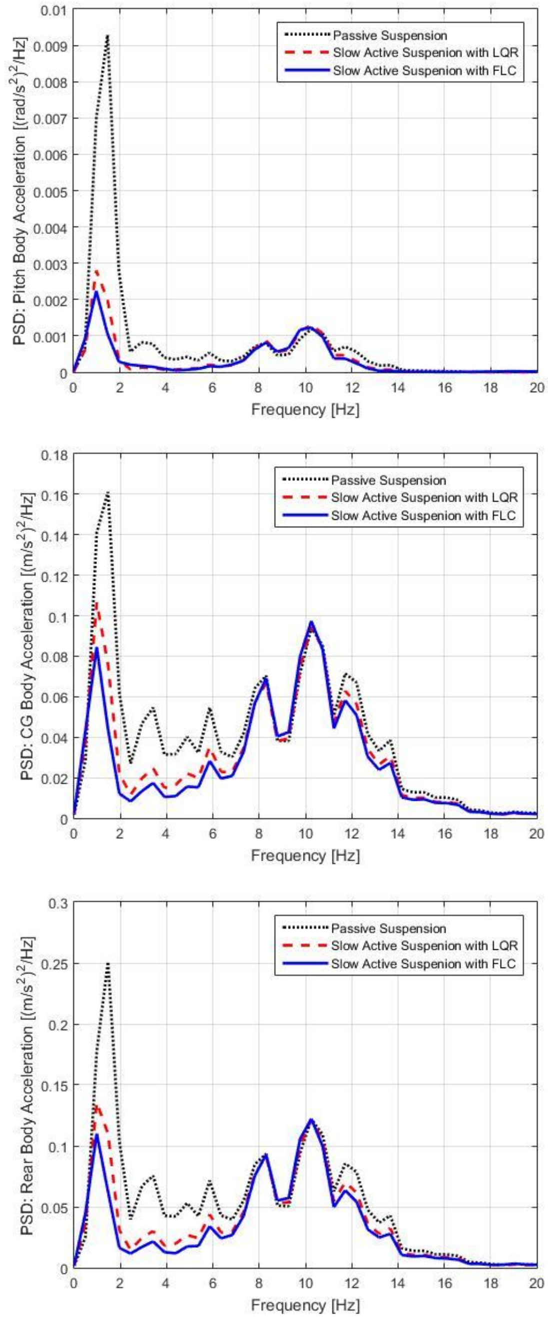
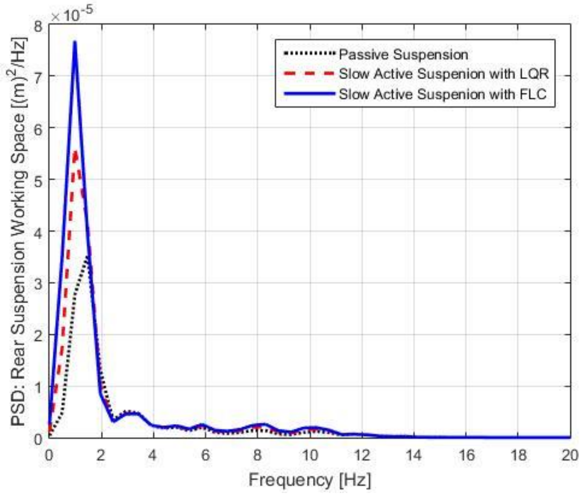


Fig. 4. Power spectral density of body C.G bounce, pitch accelerations and rear body acceleration for passive and limited bandwidth hydropneumatic active suspension with LQR and FLC using road input (class B)

Fig. 5 shows the rear suspension working space and dynamic tyre load for the passive system and limited bandwidth hydropneumatic active suspension with LQR and FLC using road input (Class C). It can be seen that, there are clear improvements around body and wheel resonances peaks for rear dynamic tyre load.



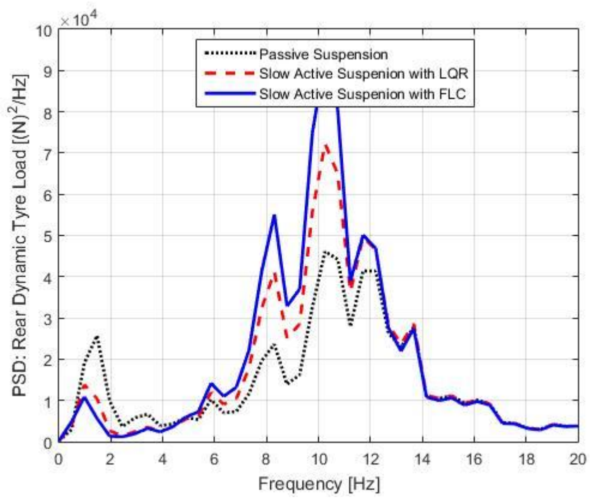


Fig. 5. Power spectral density of rear suspension working space and dynamic tyre load for passive and limited bandwidth hydropneumatic active suspension with LQR and FLC using road input (class C)

The summary of vehicle ride performance in terms of root means square of front and rear body, accelerations, pitch and vehicle body CG using road input roughness (Class (C)) is shown in Table V. The percentage improvements of pitch, front, rear body accelerations for limited bandwidth hydro-pneumatic active suspension with LQR compared with passive suspension are 33%, 12.3% and 16.1% respectively. On the other hand, the percentages reductions of pitch, front and rear body accelerations for limited bandwidth hydro-pneumatic active suspension with FLC compared the same system with LQR are 16.6%, 4% and 7.5% respectively.

Overall, the limited bandwidth hydro-pneumatic active suspension with FLC gives a significant ride performance improvement compared with passive suspension system and LBW with LQR for both roads. The percentages improvements for limited bandwidth hydro-pneumatic active suspension with FLC compared with the same system with LQR are constant for both roads (Class B and C). The values of body accelerations are depended on the type of road input, vehicle speed and the control strategy used.

TABLE V
RIDE PERFORMANCE IMPROVEMENTS WITH ROAD INPUT (CLASS C)

System Performance		Passive Suspension	LBA with LQR	LBA with FLC
Front	Acc. m/sec^2	0.8272	0.8242	0.6969
	SWS, m	0.007863	0.009138	0.009861
	DTL, N	572.2	608.8	633.5
Rear	Acc, m/sec^2	1.062	0.8907	0.8342
	SWS, m	0.008395	0.008978	0.009616
	DTL, N	593.2	642.2	682
CG Body Acc. m/sec^2		0.9213	0.7945	0.753
Pitch Acc. m/sec^2		0.1381	0.09254	0.08545

C. *Comparison of slow active suspension with optimal control and FLC in terms of power requirements*

Mean power demand for limited bandwidth hydro-pneumatic active suspensions with LQR and FLC are calculated in Table_VI. Looking first at the power demand results showed that there are little differences between these systems. The mean power demand of limited bandwidth hydro-pneumatic active suspension for rear with LQR and FLC at vehicle speed of 100 km/hr, are 45 W and 44.95 W respectively. It can be seen that, although limited bandwidth hydro-pneumatic active suspensions with FLC gives better ride performance compared with the same system with LQR, it is required nearly the same power demand for actuator. So, it is more suitable to be used in the limited bandwidth hydro-pneumatic active suspensions.

TABLE VI
RIDE PERFORMANCE IMPROVEMENTS WITH ROAD INPUT (CLASS C)

Vehicle speed (km/hr)	Power R.M.S. of Limited bandwidth active suspension with LQR ($watt$)		Power R.M.S. of Limited bandwidth active suspension with FLC ($watt$)	
80	Front	35.27	Front	35.25
	Rear	36.77	Rear	36.71
100	Front	44.09	Front	44.05
	Rear	45.01	Rear	44.95
120	Front	52.91	Front	52.87
	Rear	53.83	Rear	53.7

Comparison between limited bandwidth hydro-pneumatic active suspension with LQR and FLC at front and rear in terms of power demand using vehicle speed of 100 km/hr is shown in Fig. 6. It can be seen that, although the mean values are low, the peak values are much higher in comparison with the mean value. For more clarity, time sections for the comparison between limited bandwidth hydro-Pneumatic active suspension with LQR and FLC at front and rear in terms of power demand is presented in Fig. 7.

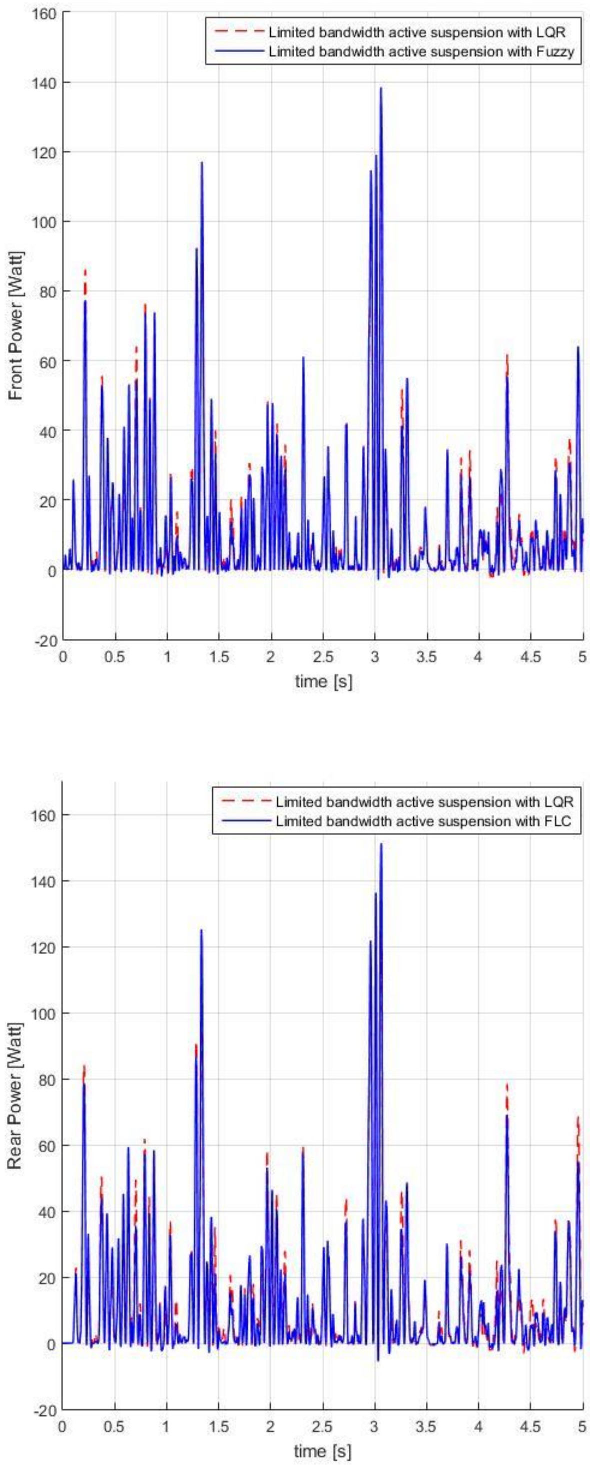


Fig. 6 Comparison between limited bandwidth hydro-pneumatic active suspension with LQR and FLC at front and rear in terms of power demand

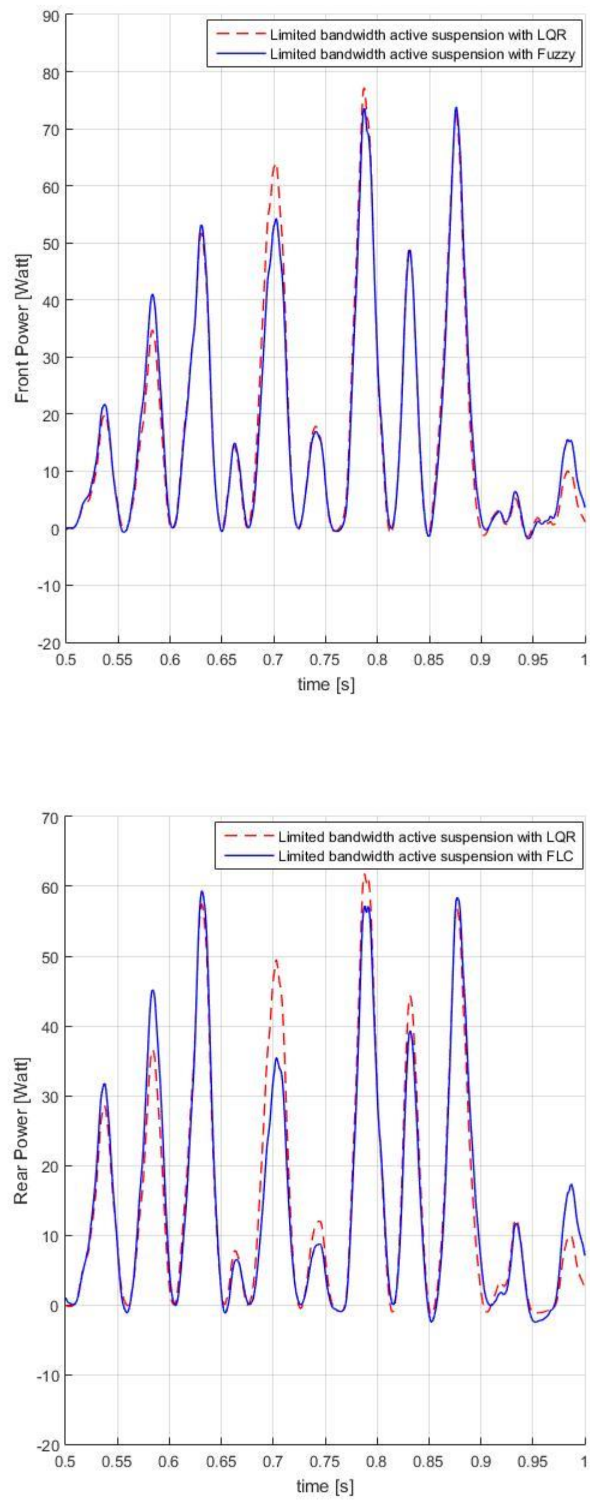


Fig. 7. More clarity comparison between limited bandwidth hydro-pneumatic active suspension with LQR and FLC at front and rear in terms of power demand

V. CONCLUSION

1. The limited-bandwidth Hydro-pneumatic active suspension with LQR gives better ride performance compared with the passive suspension system for

both roads used. The proposed control law is based on the optimal linear control theory; the system uses the more practical limited state feedback law

2. The limited bandwidth hydro-pneumatic active suspensions with FLC gives better ride performance compared with the same system with LQR, and it is required nearly the same power demand for actuator. So, it is more suitable to be used in the limited bandwidth hydro-pneumatic active suspensions.

3. The percentages reductions of pitch, front and rear body accelerations for limited bandwidth hydro-pneumatic active suspension with FLC compared the same system with LQR are 16.6%, 4% and 7.5% respectively.

References

[1] A. M. A. Soliman and D. A. Crolla, "Limited Bandwidth Active Suspension Employing Wheelbase Preview," SAE Technical Paper 2001-01-1063, 2001.

[2] S. El-Demerdash and D. Crolla, "Effect of Non-Linear Components on the Performance of a Hydro-Pneumatic Slow-Active Suspension System," IMech Part D: Journal of Automobile Engineering, vol. 210, no. 1, pp. 23-34, 1996.

[3] D. Crolla and A. Abdelhaleem, "Analysis and Design of Limited Bandwidth Active Hydropneumatic Vehicle Suspension Systems," SAE Technical Paper 2000-01-1631, 2000.

[4] M. El-Madany and A. O. Qarmoush "Dynamic Analysis of a Slow-active Suspension System Based on a Full Car Model" Journal of Vibration and Control, 16(9), January 2010.

[5] M. El-Madany, B. Al Bassam and A. Fayed, "Preview Control of Slow-active Suspension Systems," Journal of Vibration and Control, vol. 17, no. 2, pp. 245-258, 2011.

[6] M. Kaldas, K. Çalışkan, R. Henze and F. Küçükay, "Rule Optimized Fuzzy Logic Controller for Full Vehicle Semi-Active Suspension," SAE Int. J. Passeng. Cars - Mech. Systems, vol. 6, no. 1, pp. 332-344, 2013.

[7] Ramsbottom, M., Crolla, D. A., Plummer A. R. 'Robust adaptive control of an active vehicle suspension system' Journal of Automotive Engineering, Proceedings Part D, ISSN 0954-4070, Vol. 213, No D1, pp. 1-17, 1999

.

[8] J. Hu, S. Rakheja and Y. Zhang, "Tire-Road Friction Coefficient Estimation under Constant Vehicle Speed Control," IFAC-Papers OnLine, vol. 52, no. 8, pp. 136-141, 2019.

[9] A. M. A. Soliman and M. M.S. Kaldas " Semi-active Suspension Systems from Research to Mass-market – a Review" Journal of Low Frequency Noise, Vibration and Active Control, 2019.

[10] M. M. Kaldas, A. M.A. Soliman, S. A. Abdallah and F. F. Amien "Robustness Analysis of the Model Reference Control for Active Suspension System" SAE International Journal of Vehicle Dynamics, Stability, and NVH, ISSN: 2380-2162, e-ISSN: 2380-2170, Vol. 4(3), 2020.

[11] D. Crolla, G. Firth and D. Horton, "An Introduction to Vehicle Dynamics", Leeds, UK: Mechanical Engineering Department, Leeds University, 1992.

[12] G. R. Firth "The Performance of Vehicle Suspensions Fitted with Controllable Dampers" Ph. D. Thesis, Mech. Eng. Dept., Leeds University, Leeds, Uk, 1991.

[13] Kaldas, M., Soliman, A., Abdallah, S., and Amien, F., "Model Reference Control for Active Suspension System," SAE Technical Paper 2019-01-0165, 2019.

Notation

Symbol	Definition
A_{st}	Strut area
C_1	Throttle valve constant
C_d	Vehicle aerodynamic coefficient
FLC	Fuzzy logic control
$F_{sf,r}$	Front and rear passive and limited-bandwidth hydro-pneumatics active suspension
$F_{rf,r}$	Front and rear rolling resistance force
$F_{xf,r}$	Front and rear brake force
$F_{zf,r}$	Front and rear normal force
J	Performance index
H	The height of C.G from road surface
I_b	Vehicle body moment of inertia
$K_{sf,r}$	Front and rear spring stiffness
$K_{tf,r}$	Front and rear tire stiffness
$K_{1\text{-}3}$	The LQR control gains
L	Vehicle wheel base
$L_{f,r}$	Distance from C.G to front and rear axles
M_b	Vehicle body mass
M_{wf}	Front wheel mass
M_{wr}	Rear wheel mass

M_t	Total vehicle mass
P_{gd}	Demand signal pressure
P_{gsti}	Front and rear gas spring static pressure
Q_{Ai}	Front and rear actual flow rate
Q_{AD}	Front and rear desired flow rate
$q_{1\text{-}6}$	Cost function weighting parameters
RMS_{ACCi}	Front and rear body acceleration root mean square
RMS_{SWSi}	Front and rear Suspension Working Space root mean square
RMS_{DTLi}	Front and rear Dynamic Tire Load root mean square
V_{gsti}	Front and rear gas spring static volume
Z_b	Body vertical displacement at center of gravity
Z_{bi}	Front and rear body vertical displacement
$Z_{of,r}$	Front and rear road input
$Z_{wf,r}$	Front and rear wheel vertical displacement
$\ddot{\theta}$	Body pitch acceleration